(1) **The difference between local maxima/minima and absolute maxima/minima.**

If f is defined on on the closed interval $[a,b]$, local extreme values must occur at x-values inside the interior (a,b), but absolute extreme values can be at the endpoints.

When finding both local and absolute extreme values, the first thing we do is find critical points. For absolute extreme values (for functions defined on closed intervals), this is because the Extreme Value Theorem guarantees we can find absolute extreme values, so they either occur at the endpoints or in the interior, in which case they are local extreme values and therefore occur at critical points.

For local extreme values, they must occur at critical points, but not all critical points give us local max/mins. So in this case, we find the critical points and test each one using the First or Second Derivative Test.

Keep in mind that we can't always find absolute max/min values if the interval is not closed. For instance, $f(x) = x$ on $(0,1)$ has no absolute max or min. Another example is one we did in recitation: $f(x) = x - x^2/2 - xe^{-x}$ on $(-\infty,\infty)$. As $x \to \infty$, $f(x) \to -\infty$ (the term $xe^{-x} = x/e^x$ goes to 0, and $x - x^2/2 = x(1-x/2) \to \infty(-\infty) = -\infty$). Similarly, $f(x) \to \infty$ as $x \to -\infty$. So $f(x)$ takes on values arbitrarily large and arbitrarily small, so there can be no absolute max or min. But there is a local max and a local min, as we found in recitation.

(2) **When to use First vs. Second Derivative Test.**

Many times both tests can be used, and it is a matter of preference. If $f''(c) = 0$, then the Second Derivative Test cannot be used, only the First. The Second Derivative Test requires differentiating twice and plugging in a point, while the First Derivative Test requires differentiating once and plugging in two points (**if** we can factor $f'(x)$).

The reason the First Derivative Test requires only looking at two points can be explained as follows. In recitation, we considered $f(x) = \frac{x}{x^2+4}$. We calculated $f'(x) = \frac{4-x^2}{(x^2+4)^2}$, so the critical points were $c = -2, c = 2$. Looking at $c = 2$, we chose points to the left and right to be 0 and 3 and calculated $f'(0) > 0, f'(2) < 0$, so it goes from increasing to decreasing, so there is a local max at $x = 2$.

The First Derivative Test actually requires showing $f'(x) > 0$ on an entire interval of the form $(a,2)$, and that $f'(x) < 0$ on an entire interval of the form $(2,b)$. It sufficed to just check one point because if inside $(a,2)$ there were a point x with $f'(x) < 0$, then the Intermediate Value Theorem would give us a point with $f'(x) = 0$, which would give us another critical point we didn't know about. But we had already found all the critical points, so this can't happen.

To illustrate this, let $f(x) = x^7/7 - 217x^6/6 - 5320.05x^5 + 127513.5625x^4 + 6650x^3/3 - 63750x^2$. Then $f'(x) = x^6 - 217x^5 - 26600.25x^4 + 510054.25x^3 + 6650x^2 - 127500x$. It is clear that $x = 0$ is a critical point, but the other critical points are not at all obvious. If we blindly try to apply the First Derivative Test using the points -1 and 1, we get $f'(-1) = -402286.5 < 0$ and $f'(1) = 362388 > 0$, so we would think that we go from decreasing to increasing and thus have a local min at 0. But if we apply the Second Derivative Test, we calculate $f''(x) = 6x^5 - 1085x^4 - 106401x^3 + 1530162.75x^2 + 13300x - 127500$, so $f''(0) = -127500 < 0$, so we have a local max at 0!

What went wrong here? Well, when using the First Derivative Test, it turns out that $f'(x)$ can be factored as $f'(x) = x^6 - 217x^5 - 26600.25x^4 + 510054.25x^3 + 6650x^2 - 127500x = x(x+0.5)(x-0.5)(x+100)(x-300)(x-17)$, so in order to use the First Derivative Test, we had to choose test points between -0.5 and 0 and between 0 and 0.5. So choosing $-1/4$ and $1/4$ would have sufficed, but not -1 and 1.

Bottom line: if we can't factor $f'(x)$ completely, we should use the Second Derivative Test.